Radbound University

The Matrix Code Equivalence Problem and Applications

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Contemporary algebraic and geometric techniques in coding theory and cryptography
July 21th, 2022

Matrix Code Equivalence (MCE)

The Matrix Code Equivalence Problem

Matrix code $\mathcal{C}$: a subspace of $\mathcal{M}_{m \times n}(\mathbb{F}_q)$ of dimension k endowed with **rank metric**

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MCE$(k, n, m, \mathcal{C}, \mathcal{D})$:

Input: Two k-dimensional matrix codes $\mathcal{C}, \mathcal{D} \subset \mathcal{M}_{m \times n}(\mathbb{F}_q)$

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Known: Any isometry $\mu:\mathcal{C}\to\mathcal{D}$ can be written, for some $\mathbf{A}\in\mathrm{GL}_m(q),\mathbf{B}\in\mathrm{GL}_n(q)$, as

$$\mathbf{C}\mapsto\mathbf{ACB}\in\mathcal{D}$$

Known results [Couvreur, Debris–Alazard & Gaborit, 2021]

$$\mu : \mathbf{C} \mapsto \mathbf{ACB} \in \mathcal{D}, \quad \text{with } \mathbf{A} \in \mathrm{GL}_m(q) \text{ and } \mathbf{B} \in \mathrm{GL}_n(q)$$

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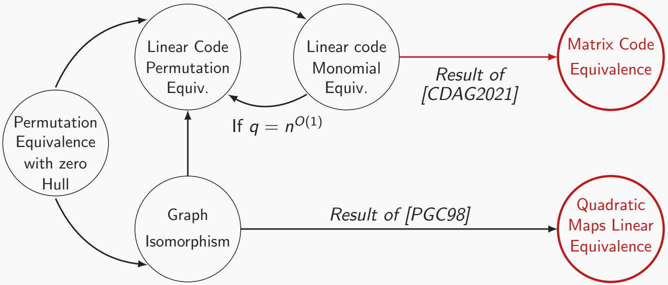

What is QMLE?

- systems of multivariate polynomials $\mathcal{P} = (p_1, p_2, \ldots, p_k)$, every p_s polynomial in N variables $x_1, \ldots, x_N$

Multivariate crypto basics

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$$p_s(x_1, \dots, x_N) = \sum \gamma_{ij}^{(s)} x_i x_j + \sum \beta_i^{(s)} x_i + \alpha^{(s)}, \qquad \alpha^{(s)}, \beta_i^{(s)}, \gamma_{ij}^{(s)} \in \mathbb{F}_q$$

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Quadratic Maps Linear Equivalence (QMLE) problem

QMLE$(N, k, \mathcal{F}, \mathcal{P})$:

Input: Two k-tuples of quadratic maps

$\mathcal{F} = (f_1, f_2, \ldots, f_k)$, $\mathcal{P} = (p_1, p_2, \ldots, p_k) \in \mathbb{F}_q[x_1, \ldots, x_N]^k$

Question: Find – if any – $\mathbf{S} \in \mathrm{GL}_N(q), \mathbf{T} \in \mathrm{GL}_k(q)$ such that

$$\mathcal{P}(\mathbf{x}) = \mathcal{F}(\mathbf{xS}) \cdot \mathbf{T}$$

Quadratic maps

$$p_s = \sum \gamma_{ij}^{(s)} x_i x_j \quad = \quad (x_1, \ldots, x_N) \underbrace{\begin{pmatrix} \gamma_{11} & \cdots & \frac{\gamma_{1N}}{2} \\ & & \\ \frac{\gamma_{N1}}{2} & \cdots & \gamma_{NN} \end{pmatrix}}_{\mathbf{P}^{(s)} \in \mathcal{M}_{N\times N}(\mathbb{F}_q)} \begin{pmatrix} x_1 \\ \vdots \\ x_N \end{pmatrix}$$

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so with $\mathbf{x} = (x_1, \ldots, x_N)$, we get $p_s(\mathbf{x}) = \mathbf{x}\mathbf{P}^{(s)}\mathbf{x}^T$
so QMLE can be rewritten in matrix form

$$\sum_{1 \leqslant r \leqslant k} \widetilde{t}_{rs} \mathbf{P}^{(r)} = \mathbf{S}\mathbf{F}^{(s)}\mathbf{S}^\top, \quad \forall s, 1 \leqslant s \leqslant k,$$

where $\widetilde{t}_{ij}$ are entries of $\mathbf{T}^{-1}$

- reduction: an MCE instance $(k, n, m, \mathcal{C}, \mathcal{D})$ results in a QMLE instance $(m + n, k, \mathcal{F}, \mathcal{P})$ with

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- solving the instance using a birthday-based algorithm $\mathcal{O}^*(q^{2/3(m+n)})$ [Bouillaguet, Fouque & Véber, 2013]

Birthday-based algorithm

- inhomogenous QMLE is solved in polynomial time

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- having a collision : $\mathbf{y} = \mathbf{xS}$, we can turn a homogenous instance into an inhomogenous instance

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- Build two lists of size $\sqrt{\kappa}$.

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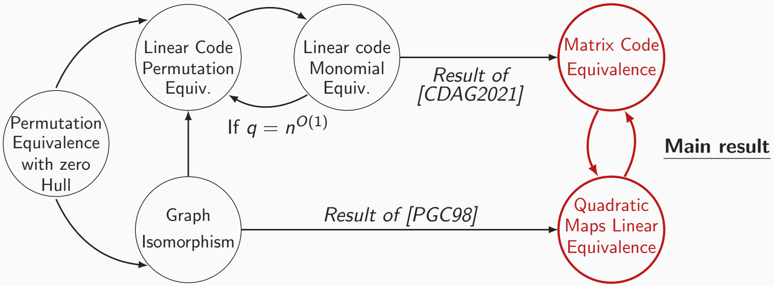
- Optimal complexity when $\sqrt{\kappa} = q^{1/3(m+n)}$.

Takeaway



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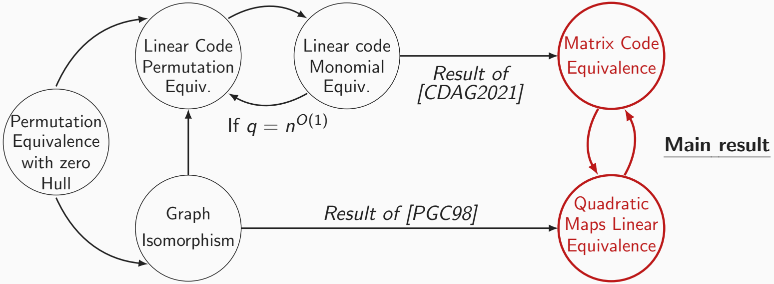

- Main result of our work: MCE **is equivalent to** QMLE
- Gives **improved upper bound** to complexity of solving MCE (w.l.o.g. assume $m \leqslant n$)
  - solvable in $\mathcal{O}^*(q^{2/3(m+n)})$ time, when $k \leqslant n + m$ can be improved to $\mathcal{O}^*(q^m)$

Matrix code equivalence:
a cryptographic group action?

MCE as cryptographic group action.

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$$\mathbf{C} \mapsto \mathbf{ACB}$$

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- one-way: our analysis show that MCE is hard.

Cryptographic Group Action: $G \times X \to X$

Given x_1 and x_2, it is hard to find an element g s.t. $x_2 = g \cdot x_1$

Building crypto from group actions

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- **Zero-Knowledge Interactive Proof of knowledge**
  - Zero-Knowledgness
  - soundness
  - can be used as identification scheme (IDS)

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- **Digital Signature via Fiat-Shamir transform**
  - F-S is a common strategy for PQ signatures
    - Dilithium, MQDSS, Picnic in NIST competition
  - From cryptographic group actions
    - Patarin's signature, LESS-FM, CSIDH, SeaSign ...

Zero-Knowledge Interactive Proof of knowledge from group actions

Let g be an element s.t. $x_1 = g \cdot x_0$.

Given x_0, x_1, the prover $\mathcal{P}$ wants to prove to the verifier $\mathcal{V}$ knowledge of g without revealing any information about it

$\mathcal{P}(x_0, x_1, g)$	$\mathcal{V}(x_0, x_1)$

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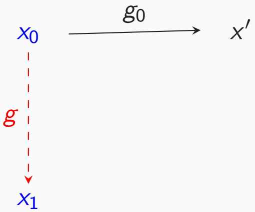

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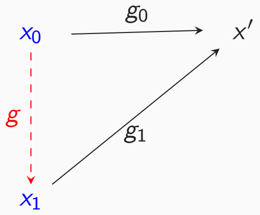

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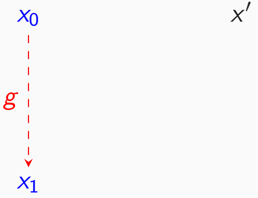

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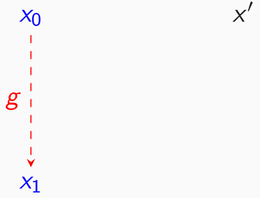

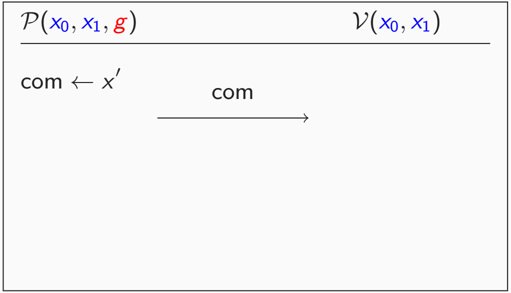

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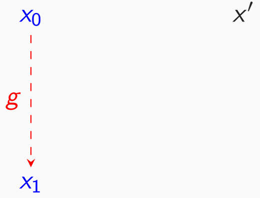

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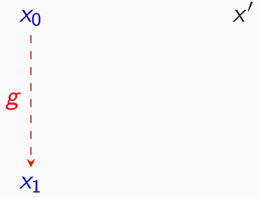

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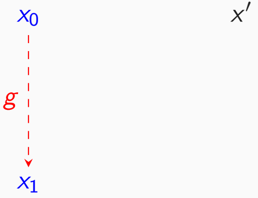

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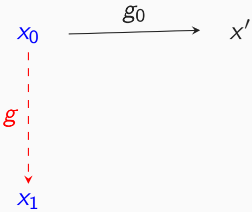

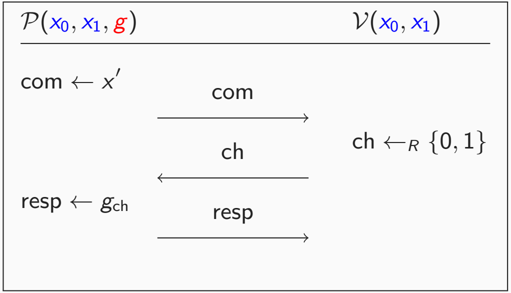

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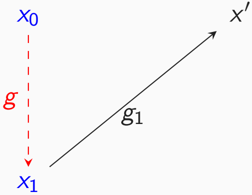

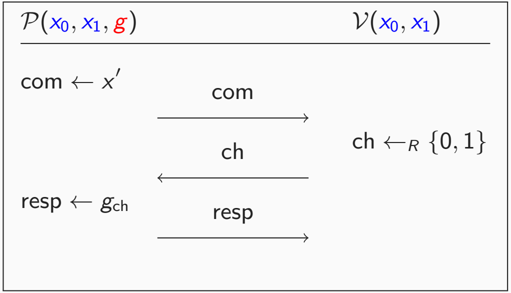

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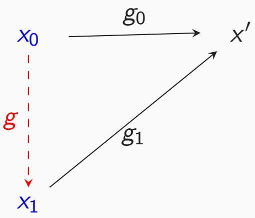

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Matrix Code Equivalence as a cryptographic primitive!

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(3) Cryptographic group action: great building block!

Matrix Code Equivalence as a cryptographic primitive!

(1) MCE is “easy to understand”

(2) Complexity linked to well-studied problem in multivariate crypto (IP)

(3) Cryptographic group action: great building block!

(4) (mathematically very interesting part of coding theory!)